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WAKE LABORATORY EXPERIMENT

By

Gordon E. Merritt

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FOREWORD

The research discussed in this report was supported by the Advanced Research Projects Agency, ARPA Order Number 1910/6-29-71, under the direction of Mr. Robert Moore. The program was monitored by Mr. Ralph D. Cooper, Program Director, Fluid Dynamics, Office of Naval Research. The author would like to thank Dr. Philip Selwyn of the Institute for Defence Analyses for helpful discussions. In addition, the contributions of Mr. Norman Kay to the experiments are acknowledged.

SUMMARY

The objective of the research is to obtain data that can be used to predict the concentration of a passive tracer released into the wake of a submarine travelling through a stratified body of water. To develop this prediction capability, information is required on the wake shape and size. Experiments have been performed in which a grid is oscillated in a stably-stratified flow to produce the steady-state counterpart of the momentumless wake of a self-propelled submarine vehicle. A pH sensitive indicator was used to produce a neutrally buoyant tracer to visualize wake development and subsequent vertical collapse as the wake turbulence is overcome by the buoyancy gradient. Measured velocity profiles in the simulated wake indicate that it is nearly momentumless and vertical temperature surveys reveal the degree of mixing in the wake.

The wake growth before and after collapse and the distance to collapse have been correlated by using power laws previously applied for the wakes of re-entry vehicles and a theoretical analysis of marine wake collapse developed under the present program. The theory is based on the balance, at wake collapse, between the potential energy of the mixed fluid and the energy of the turbulent velocity fluctuations in the wake. Experimental data available from the literature, as well as that obtained in the present program, have been used in the correlation. In its most fundamental form, the correlation gives the ratio of the vertical extent of the wake in stratified flow to that in unstratified flow as a function of the ratio of the time in the wake (measured from the body) to the Brunt-Vaisala period.

The scaling relations established for predicting stratified flow wake dimensions reveal that the important parameters are the Froude number, defined as the product of the submarine velocity and the Brunt-Vaisala period of the ocean divided by the initial wake size, and the ratio of the residence time in the wake to the Brunt-Vaisala period. Through the use of these parameters, two unique curves are obtained for estimating the horizontal width and vertical height of a wake in a stratified flow. Although enhancement of the horizontal wake growth due to vertical collapse has been observed, the area of the wake is typically less than one half that for unstratified flow.

From the measurements and analysis, equations have been developed for predicting submarine wake dimensions as a function of submarine velocity, initial wake size, ambient stratification, and time after submarine passage. The estimated submarine wake dimensions are of the correct magnitude although the equations should be regarded as preliminary since further investigation is required in several areas.

The very flat and wide wakes observed due to stratification lead to the estimate that the concentration of a passive tracer introduced into a submarine wake would remain high for long times after submarine passage.

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LIST OF SYMBOLS

A_D	initial wake area
A_w	wake area
α	ambient density gradient $\frac{1}{\rho} \partial\rho/\partial z\|_a$
β	wake density gradient $\frac{1}{\rho} \partial\rho/\partial z\|_w$
b_∞	wake diameter unstratified flow, Equation (10)
b_v	vertical wake height, Equation (12)
b_ℓ	horizontal wake width, $t/T < 1.0$, Equation (16)
b_f	asymptotic vertical wake size, Equation (13)
b_m	maximum vertical wake size, Equation (14)
b_c	horizontal wake size for $t/T > 1.0$, Equation (17)
C_D	drag coefficient of vehicle
D	initial wake diameter
D_B	diameter of vehicle
F	Froude number uT/D
F_r	Froude number $u_r T / 2\pi D$
g	acceleration due to gravity
K	constant of proportionality $\frac{b_\infty}{D} = K(\chi/D)^n$
n	exponent $\frac{b_\infty}{D} = K(\chi/D)^n$
ρ	ambient density
R_i	Richardson number, Equation (6)
T	Brunt-Vaisala period, Equation (2)
t_c	time from the vehicle to wake collapse
t	time after wake generation equal to χ/u
u'	characteristic wake turbulent velocity fluctuation $u' \approx d\,b_\infty/dt$
u	velocity of vehicle
u_r	initial wake turbulent velocity fluctuation
χ	distance in the wake from the vehicle equal to ut
z	vertical coordinate in wake from axis

LIST OF ILLUSTRATIONS

1. INTRODUCTION

The wake of a submarine travelling through a stably-stratified ocean exhibits a behavior considerably different from the more familiar wake of a vehicle moving in an unstratified environment. In both cases, the wakes grow initially with a circular cross-section. However, in the stratified flow case, mixing in the near wake destroys the ambient temperature gradient so that further downstream buoyancy forces begin to inhibit the vertical growth of the wake. Sometime after generation, the wake, which has a density gradient different from the surroundings, collapses vertically as the fluid seeks its own density level. The decrease in vertical extent of the wake enhances the horizontal growth rate so that eventually a very thin and wide wake is produced. This type of wake history in stratified flows has generated interest from the point of view of studying possible internal waves generated by wake collapse as well as tracking passive trace elements released by the submarine.

The objective of the present research program is to obtain analytical and experimental data that can be used for predicting the concentration (as a function of downstream distance) of a passive tracer released into the wake of a self-propelled body travelling through a stably-stratified medium. To achieve this prediction capability, information is required on the wake shape and size. The experimental approach utilizes a stratified flow in which a composite grid is oscillated to produce a steady-state counterpart of the momentumless wake of a self-propelled body. A pH sensitive indicator, in which local color changes are generated at the grid by electrical impulses, is used for direct observation of the wake development, diffusion and subsequent vertical collapse as the stable stratification counteracts the wake turbulence.

From the present measurements and analytical efforts along with those of other investigators, techniques are established for correlating the experimental data available on wake collapse in laboratory environments. The scaling laws produced in this manner then are used to give preliminary estimates of the growth and collapse of the wake behind a submarine travelling through a stratified ocean for comparison with full-scale data.

2. EXPERIMENTAL TECHNIQUES

A schematic arrangement of the flow system used for the experiments is given in Figure 1. The tests were performed in a water tank (210 x 10 x 10 cm) in which a stratified flow with a very nearly linear temperature gradient is generated by the mixing of streams of water at different temperatures. The degree of stratification is determined by continually measuring the vertical temperature profile through the use of small thermistor beads that traverse the depth of the water at selected locations at regular intervals.

The velocity of the flow is measured by tracers that are neutrally buoyant at every point in the fluid. This is necessary because the motion of a constant-density dye would be influenced by the density gradient, giving a distorted velocity profile. The technique[1,2] is based on the color change of an indicator solution when the hydrogen ion concentration (pH) is changed at a test point. The solution used consists of water containing 0.01% thymol blue. It is titrated to the end point by adding a small amount of hydrochloric acid and is initially bright orange in color. Fine tungsten wires stretched vertically across the water act as cathodes, and the anode is formed by a stainless steel wire along the tank bottom. Application of brief voltage pulses (90 volts) to the two electrodes reduces the hydrogen-ion concentration at the surface of the tungsten wire corresponding to a local increase in pH. Columns of dark tracer fluid are formed at the surface of the tungsten wire and move with the fluid to yield vivid velocity profiles. Typical flow speeds are about one cm/sec.

As shown in Figure 1, a wake analogous to that of a self-propelled body is generated in the water channel test section through the use of an oscillating grid approximately one quarter inch in diameter. The grid is pulsed electrically to produce a local pH - color change which traces the wake growth downstream of the grid. Measurements of the wake dimensions in both the horizontal and vertical planes are made at different distances from the grid for both unstratified and stratified flow. In the unstratified case, the wake cross-section at any point is circular. When the flow is stratified, buoyancy causes the wake, which initially grows at the same rate in all directions, to collapse in the

vertical direction and flatten out horizontally producing a very wide and thin elliptical or rectangular shape.

The determination of the wake dimensions as a function of wake position and ambient stratification is vital for the prediction of the concentration of a passive tracer released at the body. The research discussed in this report has been directed towards this objective.

3. WAKE GROWTH IN UNSTRATIFIED FLOW

The growth of a wake downstream of a body travelling through a medium in which there is no temperature gradient has been well documented in the literature. Theoretical analyses along with low-speed tests in water and hypersonic tests in ballistic ranges and wind tunnels have established relations for wake growth of the form

$$\frac{b_\infty}{D_B} = K\left(\frac{C_D x}{D_B}\right)^n \tag{1}$$

where b_∞ is the wake width (circular), D_B the diameter of the body, C_D the drag coefficient of the body and x the distance in the wake from the body. For unpropelled bodies, measurements of the turbulent wake growth rates yield values of about unity for K and 1/3 for the exponent n, in good agreement with theoretical predictions.

For the turbulent wakes of bodies with hydrodynamical self-propulsion, theoretical analyses[3] yield values for n of 1/4 for plane flow and 1/5 for axially symmetric flow. At large distances behind the body, the wake may asymptote to a constant size[4], although this has not been verified experimentally.

Various attempts[5-9] to model a submarine wake have been reported in which a two-dimensional unsteady flow is produced through the use of paddle-type mixers in a tank containing quiescent water. In these instances, the time t after mixing is equivalent to the distance in the wake x in Equation (1). It has been observed on the present research program that these measurements can be plotted in the form b_∞ / D vs. t/D for the cases in which the water is unstratified to give good correlation of the data, with b_∞ / D being proportional to $(t/D)^{1/3}$, where D is the initial diameter of the mixed region.

The value of 1/3 for the exponent is high and indicates that the mixed regions generated in the two-dimensional unsteady experiments are not strictly equivalent to those at large distances behind a self-propelled body, the wake of which would grow at a much slower rate.

In Figure 2, it is shown that tests reported by Schooley and Stewart[10] using a self-propelled model yield a wake in unstratified water that can be correlated by a relation $\frac{b_\infty}{D} = 1.0\left(\frac{x}{D}\right)^{1/3}$ obtained for hypersonic spheres. The high value of 1/3 for the exponent indicates that the wake growth in this case is determined largely by the effects of body drag and the propeller. Similarly, experiments by Naudascher[4], utilizing a jet-disc model in air to produce a steady-state counterpart of the momentumless wake of a self-propelled body, give an initial growth rate of $\frac{b_\infty}{D} = 1.5\left(\frac{x}{D}\right)^{2/5}$. This very rapid growth may be due to the influence of the jet for which the exponent n has a value of unity. For distances in the wake greater than about 20 diameters, the wake growth in Naudascher's experiment can be represented by $\frac{b_\infty}{D} \approx 1.2\left(\frac{x}{D}\right)^{1/4}$ consistent with the growth rate anticipated for a momentumless wake.

In addition to the interpretation of previous work on marine wake simulation in terms of power law growth rates, measurements have been made of the wake growth downstream of an oscillating grid in a one cm/sec flow of unstratified water. The wake growth observed using the pH-color change tracer is plotted in Figure 2 from which it is observed that the wake diameter can be expressed by the relation $\frac{b_\infty}{D} \approx 1.3\left(\frac{x}{D}\right)^{1/4}$. The wake diameter reaches a size of roughly four times the initial diameter at one hundred diameters downstream. The measurements indicate that the wake may reach an asymptotic size although further tests are required to clarify this point. The value 1/4 for the exponent in the growth law is closer to the value anticipated for the wake growth rate behind a self-propelled vehicle than the value 1/3 observed in other simulation experiments. Wake velocity profile measurements obtained using the neutrally-buoyant thymol blue tracer indicate little if any velocity defect confirming that the grid wake is nearly momentumless on the average.

Computer results from the theoretical analysis of Ko[11] are reported in the form of several different curves for Froude numbers, Fr of 0.5, 1.0 and 10.0. It is shown in Figure 2 that these can be expressed by one curve

of the form $\frac{b_\infty}{D} = 1.25\left(\frac{x}{D}\right)^{0.22}$ during the early growt' period before the effects of stratification are felt. This relation compares very well with the equation $\frac{b_\infty}{D} = 1.3\left(\frac{x}{D}\right)^{0.25}$ observed in the present experiments for unstratified flow.

The energy of the turbulence generated in the wake by the oscillating grid can be estimated by using the -verified[3] result $u' \approx db_\infty/dt$, where u' is a characteristic turbu' t velocity fluctuation, and b_∞ is the width of the wake. For the growth rate observed in the experiments, i.e. $\frac{b_\infty}{D} = 1.3\left(\frac{x}{D}\right)^{1/4}$ this yields $\frac{u'}{u} = 0.325\left(\frac{x}{D}\right)^{-3/4}$ for the rate at which the turbulence intensity decays downstream of the oscillating grid. In Figure 3 this relation for u' is shown to agree remarkably well with Naudascher's[4] hot wire anemometer measurements in air of the turbulence intensities u', v', w' on the axis behind a jet-disc model which is used to simulate the momentumless wake of a self-propelled body. This agreement confirms the hypothesis that the energy of the turbulence at different positions in the wake can be estimated from measurements of the rate at which the wake dimensions change with time, and that the turbulence decay rate can be expressed by a straightforward power law.

The results obtained from the laboratory measurements of wake growth and turbulence intensity, shown in Figures 2 and 3, can be used to estimate the size and energy of the wake of a submarine travelling through an unstratified ocean. The predicted diameter of the wake behind a typical vehicle would increase from 150 feet at two miles downstream to 225 feet at ten miles. In the same distance, the turbulent velocity fluctuations in the wake of a vehicle travelling at 10 knots would decay from approximately 2.5 cm/sec at two miles to 0.5 cm/sec at 10 miles. The wake energy represented by the square of the turbulence velocity plays an important role in determining the collapse of the wake when the flow is stratified.

4. WAKE GROWTH AND COLLAPSE IN STRATIFIED FLOW

4.1 Wake Collapse Time

The wake of a body trav- ing through an environment that is stably stratified can be profoundly influenced by the density gradient. Immediately downstream of the vehicle, the wake, which may contain fluid of nearly

constant density, will grow at the same rate in all directions, as in the unstratified flow case, (Figure 2). However, as the turbulent energy of the wake decays with increasing distance from the body (Figure 3), the restoring action of buoyancy begins to inhibit the vertical expansion of the wake and at the same time enhances the horizontal growth. At some point behind the body, the wake reaches a maximum vertical size followed by a collapse as the fluid returns under the action of gravity to the level at which its density is the same as the environment.

Many investigations[5-12], primarily two-dimensional unsteady ones, have been directed toward determining the distance or time from the body to wake collapse. A parameter used to characterize the phenomenon is the Brunt-Vaisala period T defined by

$$T = \frac{2\pi}{\left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2}} \tag{2}$$

where $\partial\rho/\partial z$ is the ambient density gradient.

Since T is the period at which a parcel of displaced fluid oscillates about its equilibrium-density position, it would be reasonable to expect that the time to wake collapse t_c would be a function of T. In a recent review by Sundaram[13], apparently conflicting results were presented in an attempt to relate t_c to T. However, it was observed on the present research program that this disagreement between results from different investigators can be attributed to the fact that Van de Watering[5,8] expresses the Brunt-Vaisala period per radian and Schooley[6,7] per cycle. In addition Schooley measures the time to collapse t_c from the start of mixing and Van de Watering from the end of mixing. As shown in Figure 4a, this can seriously influence the results since typical values for T are between 5 and 24 sec/cycle with a typical mixing time of three seconds.

When allowance is made for the different definitions of T and zero time in Figure 4a t_c/T from the two-dimensional unsteady experiments is observed to lie between 1/4 and 1/2 at low values for T where the mixing time is equal to or greater than t_c. The mixing time becomes a decreasing

fraction of the collapse time with increasing T, and the ratio t_c/T approaches a value of 1/3 at $T \approx 24$ sec/cycle, the upper limit of the measurements. For the experiments[10] with the self-propelled model in which the mixing time is very small relative to the Brunt-Vaisala period T (which itself is only three seconds), t_c/T in Figure 4a equals 1/3. Measurements reported[14,15] for towed plates are also shown to give approximately the same value for the ratio t_c/T in Figure 4a.

In the present series of experiments, the wake grows downstream of an oscillating grid that is operated continuously in a flow of thermally-stratified water as depicted in Figure 1. Possible influences of mixing time are eliminated and the collapse of the wake is not constrained by the presence of the mixer in the center as in the two-dimensional unsteady experiments. Measurements were performed for Brunt-Vaisala periods between 8 and 60 extending the range in T closer to the high values (in the hundreds) measured in the ocean. The results shown in Figure 4b indicate that the collapse time is roughly one-third the Brunt-Vaisala period over the complete range of T. It should be emphasized that the collapse time in all the experiments can be estimated only approximately since the arresting of the vertical wake growth followed by the collapse is a gradual process spread out over a considerable period of time.

4.2 Analysis of Wake Collapse

To predict the concentration of a passive tracer introduced in the wake of a self-propelled body travelling through a stratified medium, it is first necessary to establish methods for predicting the size of the wake. As part of the present program of research, a theoretical analysis of wake collapse was carried out. In the vicinity of collapse, the turbulent energy in the wake (which makes it grow) is assumed equal to the potential energy (which makes it collapse).

For a wake of rectangular cross-section with a linear density gradient, the potential energy can be approximated by $\ell_h \rho (\alpha - \beta) \ell_v^3 / 12$ and the kinetic energy by $\ell_h \rho (u')^2 \ell_v / 2$ where ℓ_v is the vertical height of the wake, ℓ_h is the horizontal width, ρ is the ambient density, u' is the

turbulent velocity fluctuation, g is the acceleration due to gravity, $\alpha = \frac{1}{\rho}\frac{\partial\rho}{\partial z}\Big|_a$ is the ambient density gradient and $\beta = \frac{1}{\rho}\frac{\partial\rho}{\partial z}\Big|_w$ is the density gradient in the wake caused by the mixing. If the wake were completely mixed, the wake would have a constant density and β would equal zero. The assumption of a rectangular wake will be shown later to be reasonable.

The representation of the potential and kinetic energies by the above expressions should be regarded as giving a parametric rather than an exact dependence. Equating the two approximate equations for the kinetic and potential energies yields the following relation for the vertical height of the wake.

$$\ell_v^2 \approx \frac{6(u')^2}{g(\alpha-\beta)} \tag{3}$$

From measurements (Figure 3), it has been established that for momentumless wakes in unstratified flow the turbulent intensity u' can be equated to the wake growth rate by $u' \approx \frac{d\ell_\infty}{dt}$. Combining this relation with Equation (1), and taking $x = ut$ where u is the vehicle velocity, yields

$$\frac{u'}{\ell_\infty} = \frac{n}{t} \tag{4}$$

Combining Equations (4), (3) and (2) gives

$$\frac{\ell_v}{\ell_\infty} \approx \frac{0.40\, n}{\left(\frac{t}{T}\right)\left(1 - \frac{\beta}{\alpha}\right)^{1/2}} \tag{5}$$

In other words, the ratio of the vertical width ℓ_v of the wake under stratified flow conditions to the width ℓ_∞ when the flow is not stratified can be expressed as a simple function of t/T (the ratio of time $t = x/u$ between the body and the wake location to T the Brunt-Vaisala period). The degree of mixing in the wake enters in the term $(1 - \frac{\beta}{\alpha})^{1/2}$.

A physical understanding of the t/T scaling can be obtained by noting that interactions involving flow stratification are often characterized by the local Richardson number Ri which is defined in Equation (6).

$$R_i = \frac{t_\infty^2}{(u')^2}\,\frac{g}{\rho}\,\frac{\partial\rho}{\partial z} \tag{6}$$

The parameter t/T is directly related to R_i since Equation (6) leads to

$$R_i = \left(\frac{2\pi}{n}\,\frac{t}{T}\right)^2 \tag{7}$$

Essentially the Richardson number, R_i, represents the ratio of the potential energy to the turbulent energy. Using Equation (7) in Equation (5) reveals that $\ell_v/\ell_\infty \approx 0.8\pi/(R_i)^{1/2}(1-\beta/\alpha)^{1/2}$. Hence, as physically anticipated, an increase in potential energy (higher Richardson number) decreases the vertical extent of the wake while an increase in turbulence (lower Richardson number) increases the vertical wake size.

4.3 Measurements of Degree of Mixing in Wake

An important parameter revealed in the analysis of wake collapse, Section 4.2, is the degree of mixing in the wake, β/α. The effect of mixing on the time to wake collapse t_c can be determined by taking $\ell_v/\ell_\infty = 1.0$ in Equation (5). This yields

$$\frac{t_c}{T} = \frac{0.40\,n}{\left(1-\frac{\beta}{\alpha}\right)^{1/2}} \tag{8}$$

In Figure 5, the ratio t_c/T is plotted against the degree of mixing β/α for n equal to 1/3 and 1/5. For the completely mixed case, ($\beta/\alpha = 0$, and $n = 1/4$), $t_c/T \approx 0.10$ whereas for the slightly mixed case, ($\beta/\alpha = 0.9$), $t_c/T \approx 0.32$. This is the value observed in Figure 4 that best correlates the available experimental data on wake collapse time. As shown in Figure 5, an almost identical variation in t_c/T with β/α has been observed in the computer results from Ko's[11] theoretical analysis in which the value of $\beta/\alpha = 0.9$ also was observed to best fit the measurements.

To determine the degree of mixing in the wake as a function of downstream distance, measurements of the vertical temperature profiles both inside and outside of the wake have been obtained using small thermistor beads that traverse the flow. The results are presented in Figure 6 for a Brunt-Vaisala period of 16. The ambient profile given in Figure 6a shows a temperature gradient of roughly 3/4 ° C/cm. In Figure 6b, a temperature probe at a fixed vertical position reveals temperature fluctuations in the wake of about 0.1° C compared with the smooth profile obtained in the undisturbed stream when the grid is not oscillated. This fluctuation is equivalent to roughly 10 to 20% of the ambient temperature difference across the wake at this point. The probe is located just inside the upper edge of the wake and the drop in temperature shown in Figure 6b when the grid is oscillated confirms that the water is being mixed. This can be seen in the wake temperature profile in Figure 6c in which a wake of nearly constant temperature has been generated near the grid. As shown in Figures 6d, e, and f, the wake temperature profile returns closer to the ambient one with increasing distance downstream of the grid although large scale motions hinting at interval waves are observed.

From measured profiles such as those given in Figure 6, the degree of mixing β/α in the wake was calculated. The results are presented in Figure 7 as a function of time after wake generation. Although considerable uncertainty exists, it is apparent that by the start of collapse at about $t/T \simeq 1/3$ (Figure 4), β/α is approaching a value not much less than unity. This is consistent with analytical observations discussed in Section 4.2 requiring a value of $\beta/\alpha = 0.9$ to fit theory to experiment.

4.4 Measurements of Wake Collapse

In addition to the analysis of marine wake collapse and the correlation of earlier results carried out on the present contract, measurements of wake growth and collapse were obtained for Brunt-Vaisala periods up to 60. This compares with a value of 3 for the self-propelled model experiments[10] and 6 to 24 for the two-dimensional unsteady experiments[6-9] that are influenced by the mixing time and the presence of the mixer at the center of the turbulent region. The present experiment was run continuously with the temperature

gradient allowed to decrease gradually with increasing time so that T varied from about 8 sec/cycle at the start of the experiment to 60 at the end, several hours later. During this period, the wake collapse was observed to move downstream as predicted by the relation $t_c/T \simeq 1/3$.

Measurements of the vertical and horizontal growth of the wake, obtained using the pH-color change technique and the experimental configuration shown in Figure 1, are presented in Figure 8 for a Brunt-Vaisala period of 55. The measurements are normalized by D, the initial wake diameter which is approximately equal to the grid diameter. From the grid to an x/D of 15 which corresponds to a value for t/T of 0.15, the vertical b_v and horizontal b_h size of the wake are approximately equal to the diameter b_∞ measured in the unstratified flow case, Figure 2. Further downstream, the vertical size of the wake reaches a maximum of about three diameters and then decreases as the wake collapses. This causes the horizontal extent of the wake to grow at a faster rate than observed in the unstratified flow case. The vertical size of the wake appears to reach an asymptote of roughly 1.7 times the initial diameter by a time of about one Brunt-Vaisala period.

The cross-sectional area of the wake calculated from the measurements in Figure 8 is shown in Figure 9 as a function of time after wake generation. The wake area A_w is normalized by the initial area of the wake A_D. For unstratified flow, the circular wake grows according to the relation $\frac{A_w}{A_D} = (1.3)^2 \left(\frac{ut}{D}\right)^{1/2}$ obtained from the measurements in Figure 2. For stratification, an assumption must be made concerning the shape of the wake. Ko[11] assumes an elliptical wake. In Figure 9, this assumption is shown to lead to an unrealistic decrease in wake area just after collapse at $t/T = 1/3$. The same behavior is observed when the wake area is calculated for measurements reported by other investigators[5-10]. A rectangular wake will give values for A_w which are too high at collapse but which will likely be realistic at later time.

The most probable variation of wake area with time is shown by the dotted line in Figure 9. The initially circular wake starts to collapse in the form of an ellipse but quickly approaches a rectangular cross-section.

initial growth rate only slightly so that $\ell_v / \ell_\infty \approx 0.9$ to 1.0 before collapse at $t/T \simeq 1/3$. The density gradient inside the wake at this time and not that at $t = 0$ determines the wake collapse. From Figure 9, it is seen that the wake area at $t/T = 1/3$ is roughly ten times the initial area. At $t = 0$, the wake may well be completely mixed with $\beta / \alpha = 0$. However, with growth, the entrainment of such a large quantity of ambient fluid into the wake would be expected to produce a density gradient at collapse not much different from the ambient gradient, as observed.

To estimate the vertical extent of a wake during the collapse phase, Equation (5) can be combined with the measured growth rate in unstratified flow $\frac{\ell_\infty}{D} = K(x/D)^n$ to yield

$$\frac{\ell_v}{D}\left(\frac{1}{F}\right) = \frac{0.40\,Kn}{\left(1-\frac{\beta}{\alpha}\right)^{1/2}}\left(\frac{x}{D}\right)^{n-1} \tag{9}$$

where $F = \frac{uT}{D}$ is a Froude number defined using the submarine velocity u, the ambient Brunt-Vaisala period T, and the initial wake diameter D. Comparison with Ko's[11] definition of a Froude number $F_r = u_r T/2\pi D$ where u_r, the initial turbulent intensity is taken by Ko to be equal to $0.25\,u$, gives a relation between the two Froude numbers of $F \approx 25\,F_r$.

Following Equation (9), $\frac{\ell_v}{D}\left(\frac{1}{F}\right)$ is plotted against x/D in Figure 11. For comparison, Ko's[11] theoretical curves from wake collapse on downstream for F_r of 0.5, 1.0 and 10 are also given in the figure. When plotted in this fashion, Ko's results for different Froude numbers fall into one curve given by $\ell_v / D(F) = 0.57\left(\frac{x}{D}\right)^{-4/5}$. For the present analysis, as shown in Figure 11, this corresponds to Equation (9) with $K = 1.3$, $\beta/\alpha = 0.94$, and $n = 1/5$.

A simple scaling law for the post collapse behavior of the wake is thus obtained from the two analyses when the vertical size of the wake is divided by the Froude number, F, which is proportional to the ratio of the turbulence energy to the potential energy. In physical terms, at a given wake position X/D, the vertical extent of the wake, ℓ_v / D, which is directly proportional to the Froude number, F, increases as the wake turbulence increases with submarine velocity u and decreases as the potential energy (D/T) increases.

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In the use of Equation (9) just described, there is some uncertainty about the values for K and n while the degree of mixing in the wake β/α needs to be investigated further.

In Figure 12, a comparison is made between predictions of the vertical dimensions of the wake using Ko's[11] computer analysis and the power law growth rates developed on the present program. The vertical size is plotted against wake position X/D for F = 250. The Froude number F is inversely proportional to the square root of the Richardson number. For typical ocean conditions and submarine dimensions, a value of 250 for F corresponds to a submarine speed of roughly five knots.

The curve for unstratified flow, $\ell_\infty/D = K(x/D)^n$ plotted in Figure 12 gives the extent of the wake up to collapse. A family of curves obtained from Figure 11 for different degrees of mixing β/α gives the size of the wake after collapse. Comparing the $\beta/\alpha = 0.94$ curve with Ko's[11] analysis, shown by the dotted line in Figure 12, excellent agreement is obtained except in the immediate vicinity of collapse. At the intersection of the unstratified flow and post collapse curves, a maximum difference of roughly 12% is observed. Clearly, a fairing of the two power-law curves would give closer agreement with Ko's result in this region. The important point is that both analyses give the same decay rate of vertical size with distance in the wake.

To estimate the size of a submarine wake, it is convenient to express the dimensions in terms of the time after submarine passage. Before collapse, the growth of the circular wake given by $\frac{\ell_\infty}{D} \approx K(x/D)^n$ can be expressed in the form

$$\frac{\ell_\infty}{D} = K(F)^n\left(\frac{t}{T}\right)^n = 1.3\,(F)^{1/4}\left(\frac{t}{T}\right)^{1/4} \tag{10}$$

Combination of Equation (10) with the analytical estimate of the vertical extent of the wake during collapse given by Equation (5) and confirmed by the experimental data in Figure 10 yields an expression for ℓ_v/D in terms of t/T.

$$\frac{\ell_v}{D} = \frac{.40\, n K (F)^{n}}{(1 - \beta/\alpha)^{1/2}} \left(\frac{t}{T}\right)^{n-1} \tag{11}$$

Equation (11) gives the vertical wake height normalized by the initial wake diameter as a function of the initial Froude number, the degree of mixing in the wake, two parameters determined from the wake growth rate in unstratified flow, and the time after generation normalized by the ambient Brunt-Vaisala period. Substitution of the experimental values of $K = 1.3$, $n = 1/4$ (Figure 2) and $\beta/\alpha \approx 0.93$ (Figure 10) into Equation (11) yields

$$\frac{\ell_v}{D} = 0.5\,(F)^{1/4} \left(\frac{t}{T}\right)^{-3/4} \tag{12}$$

4.5.2 Vertical Asymptote and Maximum Extent

The vertical size of the wake can be estimated using Equation (12) between collapse and $t/T \approx 1.0$ at which point measurements such as those in Figures 8 and 10 indicate that the wake vertical size may reach an asymptote ℓ_f. Putting $t/T = 1.0$ in Equation (12) gives

$$\frac{\ell_f}{D} \simeq 0.5\,(F)^{1/4} \tag{13}$$

Equation (13), which requires further verification, gives a relation for the asymptotic vertical wake size ℓ_f in terms of the initial size and Froude number. For the conditions of the present experiment, $u = 1.1$ cm/sec, $D = 0.6$ cm, and $T = 55$ sec, $F = \frac{UT}{D} = 100$. As shown in Figure 13, Equation (13) gives an asymptotic wake height ℓ_f/D of 1.6 compared with a measured value of 1.7 in Figure 8. For Schooley and Stewart's[10] self-propelled model experiments in which $u = 45$ cm/sec, $D = 2.2$ cm, and $T = 2.3$ sec, Equation (13) gives a value of 1.4 for ℓ_f/D equal to that measured.

A similar relation can be obtained for the maximum vertical extent ℓ_m of the wake by substituting $t_c/T \approx 1/3$ from Figure 4 into

Equation (10) and noting from Figure 10 that $\ell_m / \ell_\infty \approx 0.9$ to give

$$\frac{\ell_m}{D} \approx 0.9 \; (F)^{1/4} \tag{14}$$

For the present experiment, Equation (14) as shown in Figure 13 gives $\ell_m / D \approx 2.8$ compared with a measured value of 2.9 while for Schooley and Stewart's tests the measured and estimated values are both 2.4.

Ko's[11] analysis is shown in Figure 13 to give the same variation of ℓ_m / D with Froude number, F, although the values of ℓ_m / D are slightly lower than those measured. In addition, Ko's analysis predicts values of ℓ_f / D less than those measured in the region where an asymptote appears to be reached. This could account in part for Ko's analysis overpredicting the horizontal extent of the wake.

Van de Watering et al[8] report measurements of ℓ_m / D and ℓ_f / D in terms of the initial wake growth rate. Use of Ko's assumptions that $u'/u = 0.25$ initially for Van de Watering's measurements gives values of (F) between 2 and 10, which would correspond to submarine velocities of less than one knot. Van de Watering's measurements also yield the $(F)^{1/4}$ variation given in Figure 11 in which his measurements at the highest value of $F = 10$ are shown to agree well with Equations (13) and (14).

4.5.3 Horizontal Width

As the wake collapses vertically, the horizontal growth rate is enhanced so that the horizontal width ℓ_h becomes greater than the diameter in unstratified flow ℓ_∞. The ratio ℓ_h / ℓ_∞ from the present as well as other investigations[6-11] is correlated quite well in Figure 10 by the parameter t/T and reaches a value of about 1.6 at $t/T = 2$ compared with the vertical size $\ell_v / \ell_\infty \approx 1/3$. In other words, the wake width at this point is roughly five times the wake height. Ko's theory is shown in Figure 10 to overpredict the horizontal wake size for t/T greater than about 0.7. This is probably due to Ko's assumption of an elliptical wake, although the presence of the sidewalls ma, ave restricted the horizontal growth in some of the previous experiments[10].

To predict the horizontal extent of a wake during collapse, use can be made of the correlation of measurements in Figure 10. The data can be fitted by

$$\frac{\ell_h}{\ell_\infty} \approx 1.55 \left(\frac{t}{T}\right)^{1/4} \tag{15}$$

This expression when combined with the unstratified growth rate, Equation (10), gives for K = 1.3 and n = 1/4

$$\frac{\ell_h}{D} \approx 2.0\,(F)^{1/4}\left(\frac{t}{T}\right)^{1/2} \tag{16}$$

Equation (16) gives the horizontal enhancement of the wake growth rate during the period when the wake is collapsing vertically. If, as postulated, a vertical asymptote is reached at about $t/T \approx 1.0$, it is reasonable to expect that the horizontal growth rate would be reduced. In this region, the wake width ℓ_c is assumed to increase at the same rate as in unstratified flow with n = 1/4 but with K = 2.0 because Equation (16) must be matched at t/T = 1.0. This gives

$$\frac{\ell_c}{D} = 2.0\,(F)^{1/4}\left(\frac{t}{T}\right)^{1/4} \tag{17}$$

5. ESTIMATES OF SUBMARINE WAKE DIMENSIONS

In the past, wake growth rates measured in ballistic ranges and in small scale laboratory investigations of unstratified flow over fixed models have been used successfully to predict the wake growth for full-scale unpowered reentry vehicles. In a similar manner, the laboratory measurements of grid generated and model wakes can be used to predict the wake growth behind a self-propelled submarine vehicle using Equation (10) up to collapse ($t_c/T \approx 1/3$) and Equations (12) - (17) downstream of collapse. With the best values presently available for the uncertain parameters used in those equations (K = 1.3, n = 1/4, β/α = 0.93), it is possible to estimate submarine wake dimensions for different speeds, initial wake sizes, and ambient

stratification. In addition, the effect of varying the parameters K, n, and β / α determined from theory and experiment could be calculated.

Referring to Equations (12) and (16), it is observed that both the vertical, ℓ_v, and horizontal, ℓ_h, dimensions of the wake vary weakly with the submarine velocity u being proportional to $u^{1/4}$. As a consequence, measurements of wake dimensions at equal times after submarine passage would not change very much with submarine velocity. On the other hand, the vertical extent of the wake during collapse is very sensitive to the ambient stratification being directly proportional to the Brunt-Vaisala period T, although the asymptotic height only varies with $T^{1/4}$.

The scaling relations established in Section 4 from experimental data for wake growth and collapse reveal that the important parameters are the Froude number, F, and the ratio t/T of the residence time in the wake to the Brunt-Vaisala period. This is demonstrated in Figure 14 where the wake dimensions ℓ/D divided by $(F)^{1/4}$ are plotted against t/T. In this manner, two unique curves are obtained for estimating the horizontal width and vertical height of a wake in stratified flow. Verification of these curves or similar ones possibly modified by different values for the constants and exponents must rest ultimately on comparison with submarine data.

From the equations and curves for wake growth in Figure 14, unique curves can also be determined for the wake area. As shown in Figure 15, this is accomplished by plotting $A_w/A_D\,(F)^{1/2}$, the wake area normalized by the initial area and divided by the square root of the Froude number, against the time after wake generation. The curves show that when the flow is stratified, the inhibition of vertical entrainment after collapses reduces the wake area to approximately one half the area anticipated for unstratified flow. The assumption of an elliptical wake during collapse is seen in Figure 15 to lead to an unrealistic decrease in wake area with time. This can be avoided by assuming that the wake becomes nearly rectangular in shape soon after collapse.

As shown in Figure 16, the equations developed (Figure 14) can be used for preliminary estimates of the growth of the wake behind a submarine travelling through a stratified ocean. Parameters chosen for the calculation are a submarine velocity U of 12 knots, an initial wake size D of 25 feet

and a Brunt-Vaisala period T of 12 minutes. Referring to Figure 16, the wake grows initially in a conical manner and reaches a diameter of about 100 feet at two minutes. A maximum vertical height of about 110 feet is reached between three and five minutes followed by a collapse to the asymptotic vertical height of approximately 60 feet at fifteen minutes. During this collapse phase, the horizontal growth rate is enhanced so that by fifteen minutes the wake is roughly 260 feet wide. By forty minutes after wake generation, which corresponds to a position nine miles downstream of the submarine, the wake is estimated to be approximately 300 feet wide and 60 feet high. From Figure 3, the turbulent velocity in the wake at this time is estimated to be of order 0.5 cm/sec so that subsequent dispersion of the wake constituents would depend on the background ocean turbulence.

Estimated extremes in wake dimensions can be calculated from the equations and curves developed in Figure 14. The horizontal and vertical sizes forty minutes after submarine passage are plotted in Figure 17 against submarine velocity for different oceanic stratification. The initial wake diameter is taken as twenty-five feet. For a submarine moving very slowly (u = 2 knots) through a highly stratified ocean (T = 4 min.), the wake height would be 30 feet. In contrast, a fast submarine (u = 25 knots in a slightly stratified ocean (T = 40 min.) would produce a wake 100 feet high. A wake 500 feet wide would be produced for u = 25 knots and T = 4 min. whereas the width would be only 150 feet for U = 2 knots and T = 40 min.

With an estimate of the size and shape of the wake available, preliminary predictions of the concentration of a passive tracer introduced at the submarine could be made. In a given cross-section, it would be reasonable to anticipate maximum concentrations at the wake center with some form of exponential decay in the horizontal direction to zero at the wake edge and little variation in the vertical direction. Based on the estimates of wake area, high values of tracer concentration would be observed at large distances from the submarine.

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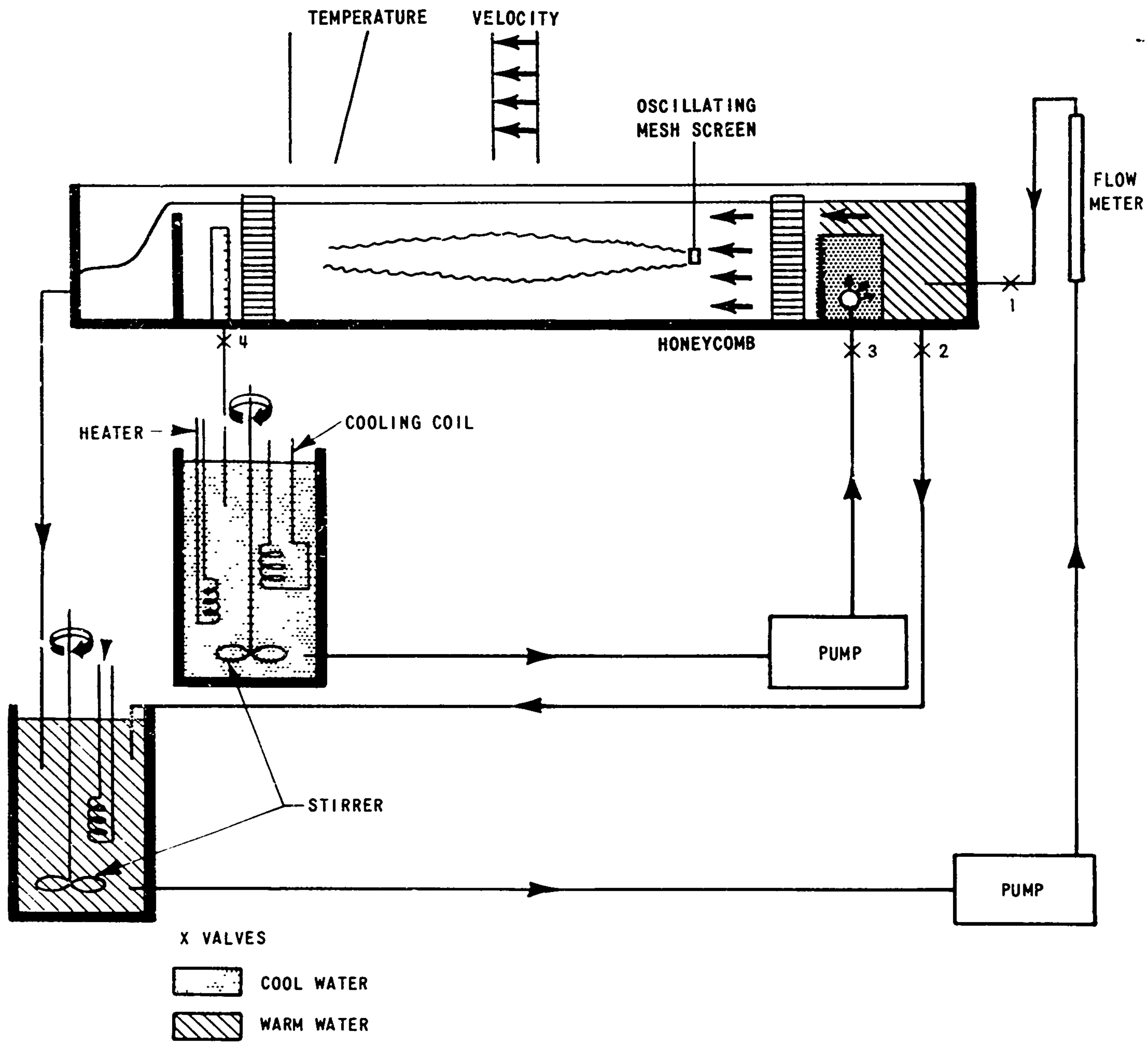

Figure 1 SCHEMATIC ARRANGEMENT OF FLOW SYSTEM

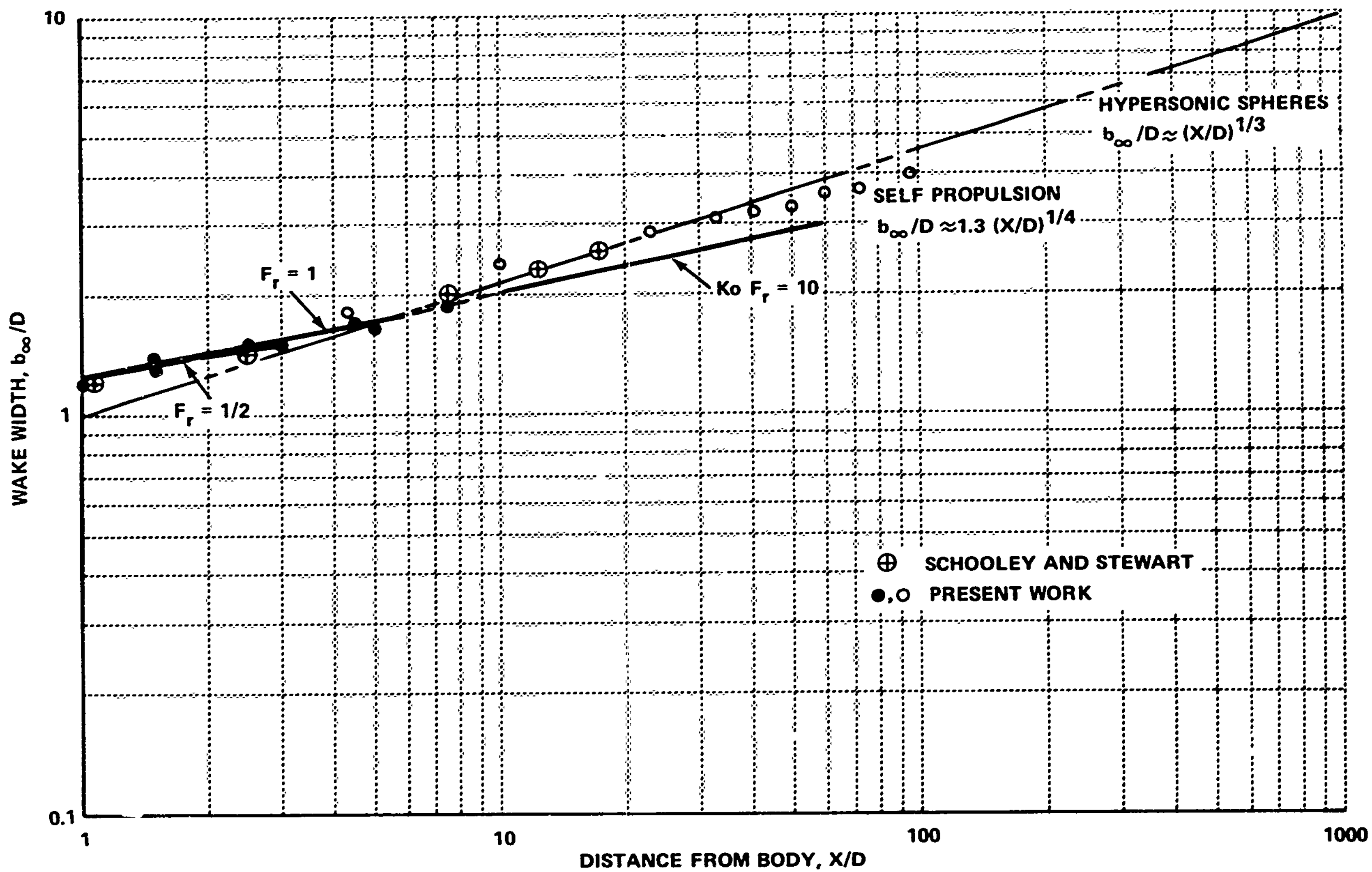

Figure 2 GROWTH RATES IN UNSTRATIFIED FLOW

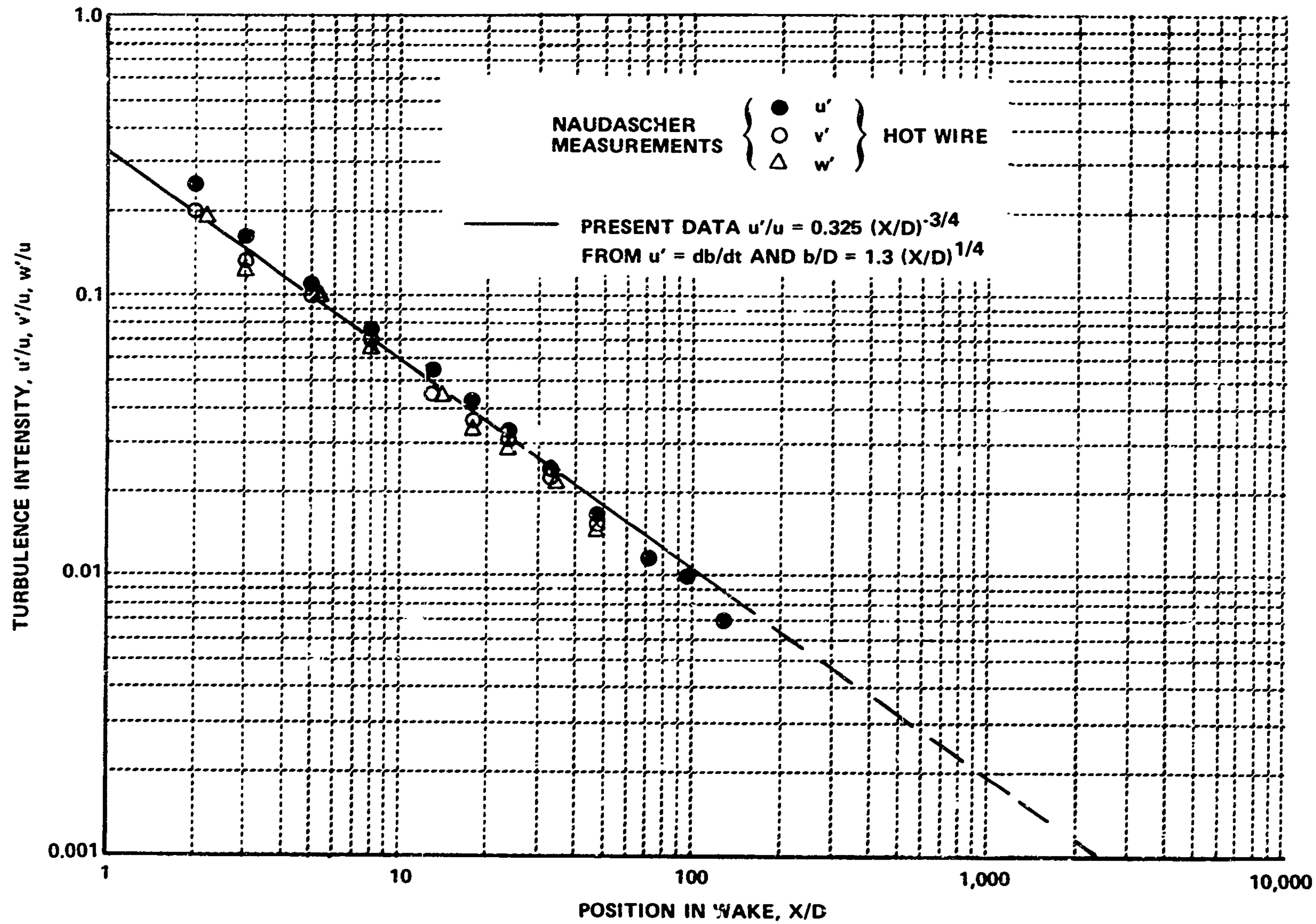

Figure 3 DECAY OF TURBULENCE INTENSITY (UNSTRATIFIED FLOW)

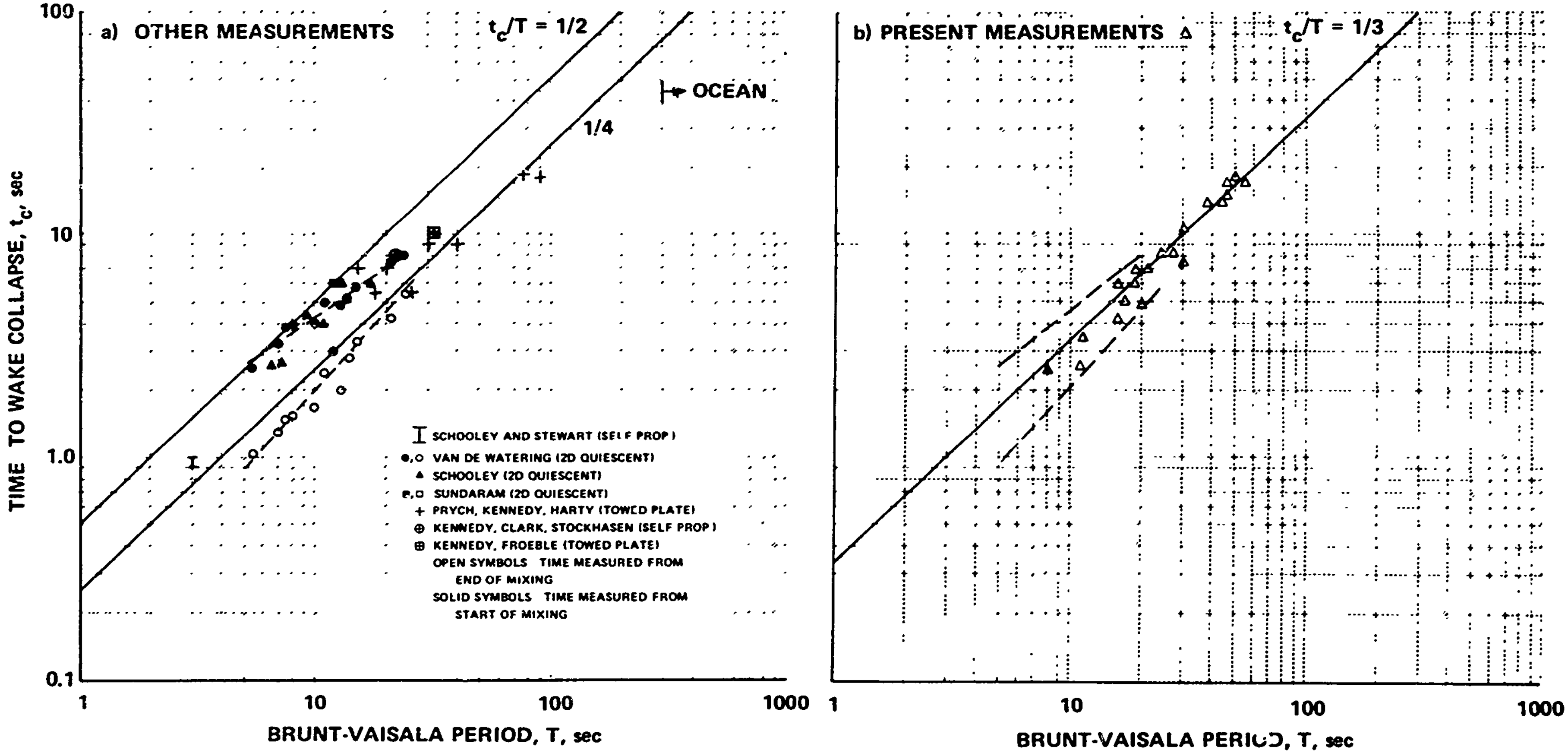

Figure 4 CORRELATION OF WAKE COLLAPSE TIME WITH BRUNT-VAISALA PERIOD

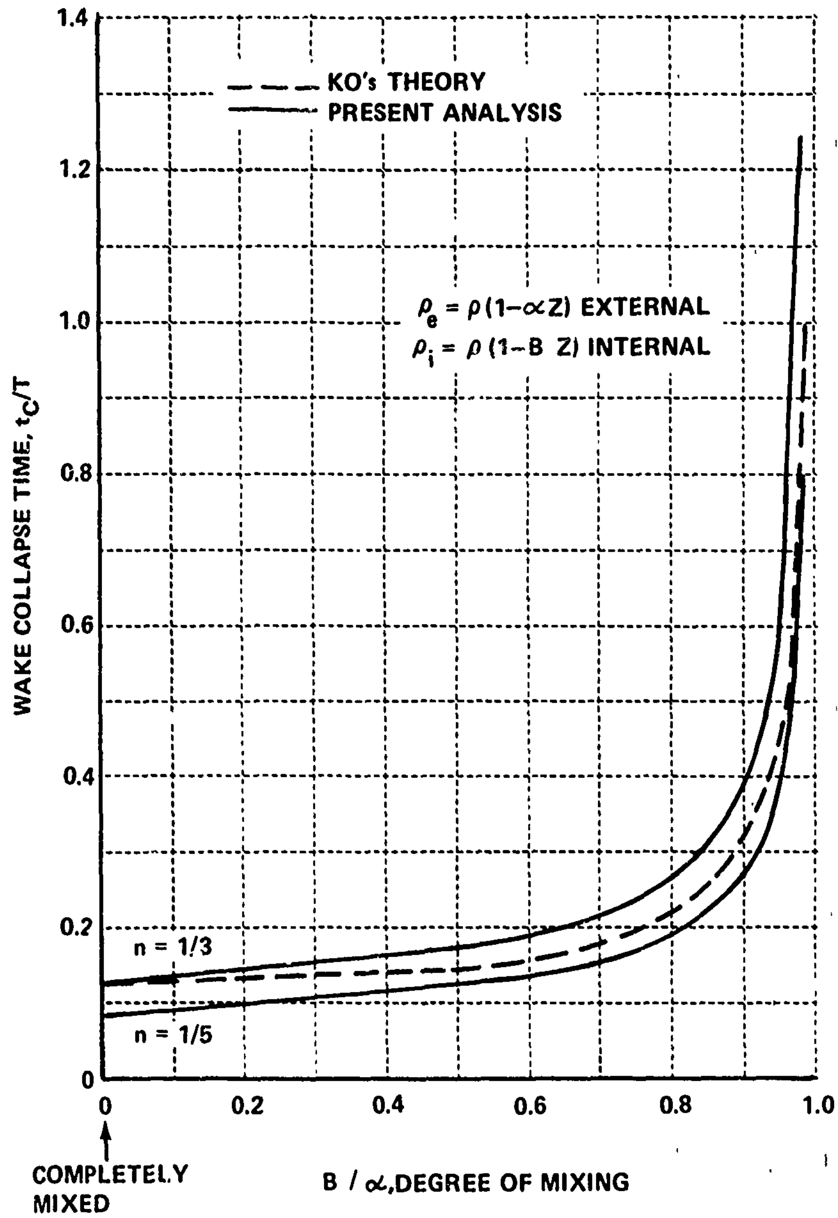

Figure 5 EFFECT OF MIXING ON WAKE COLLAPSE TIME

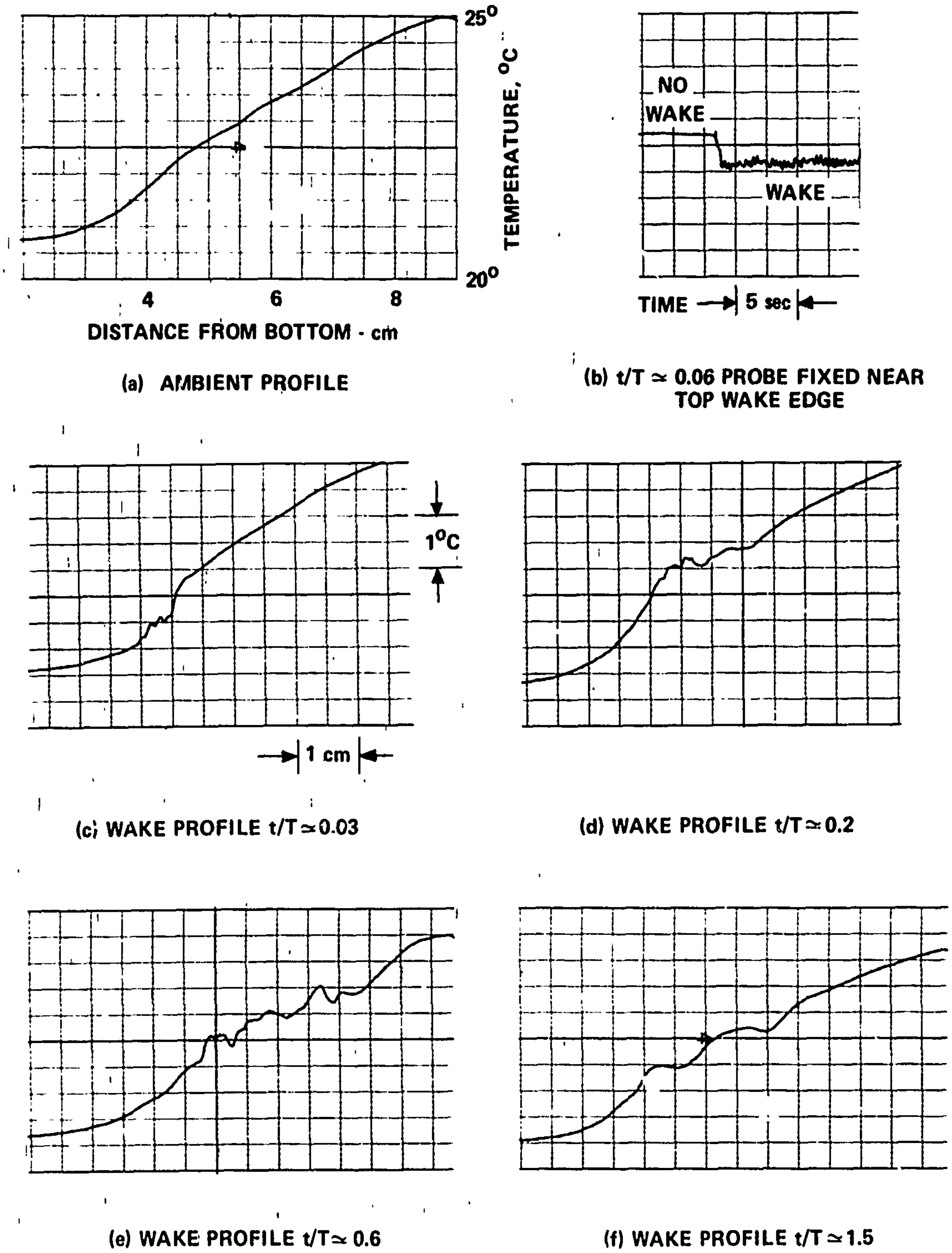

Figure 6 MEASURED TEMPERATURE PROFILES T ~ 16

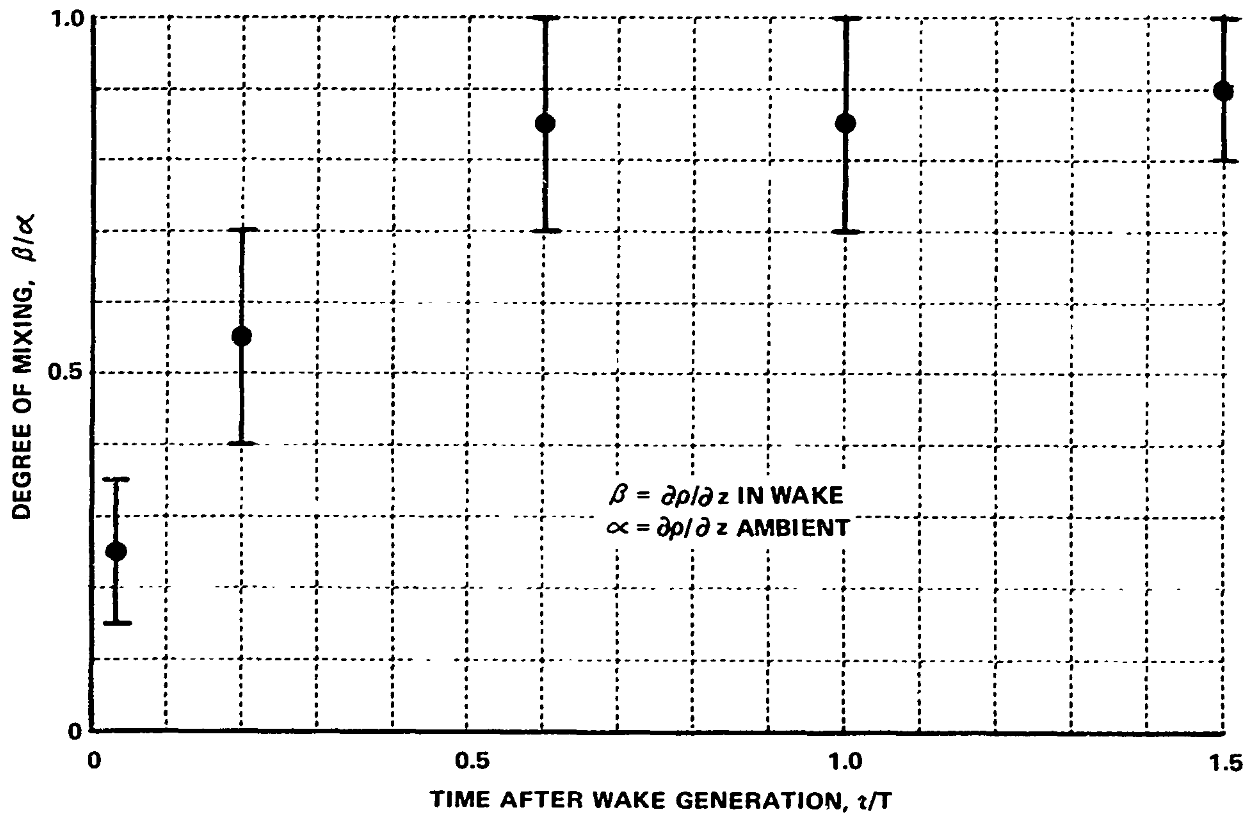

Figure 7 MEASURED WAKE DEGREE OF MIXING, T ~ 16

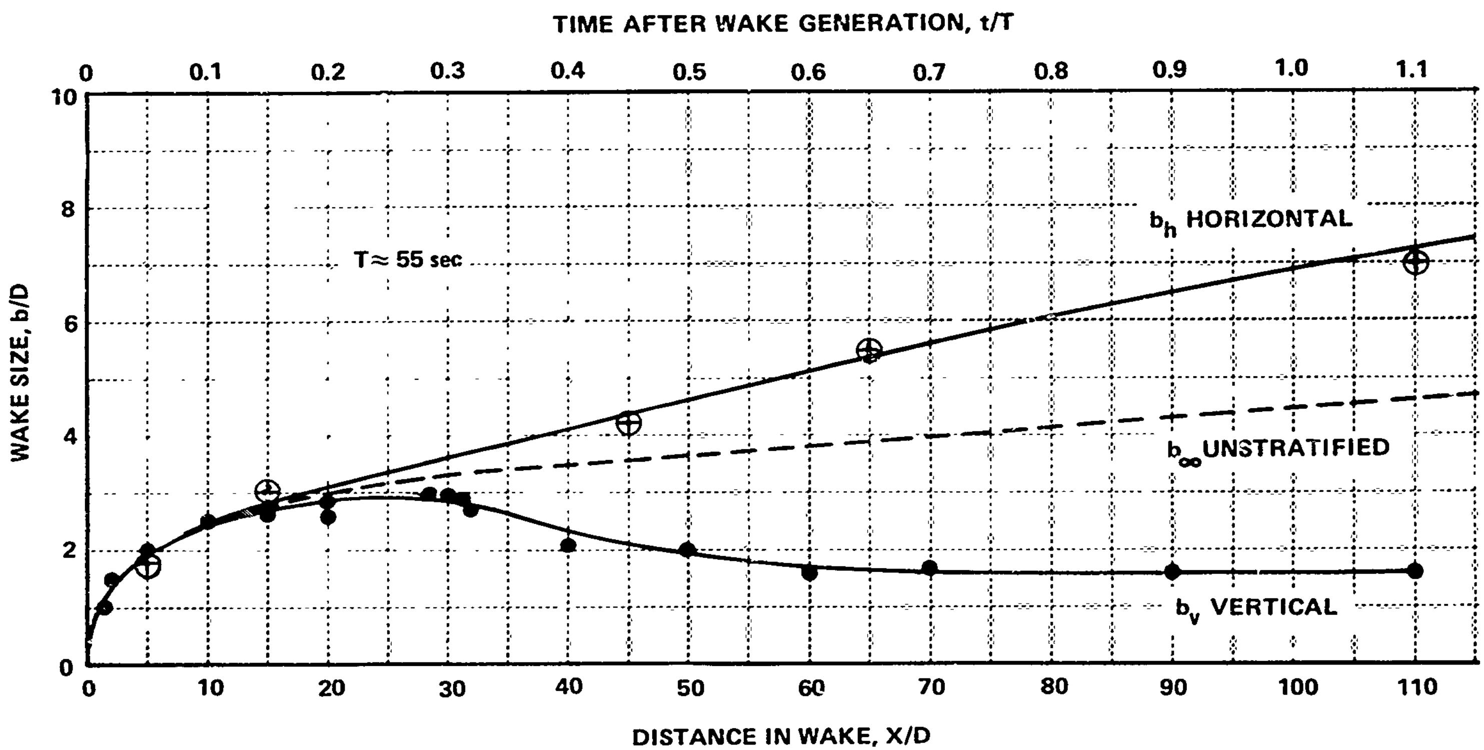

Figure 8 MEASURED WAKE GROWTH

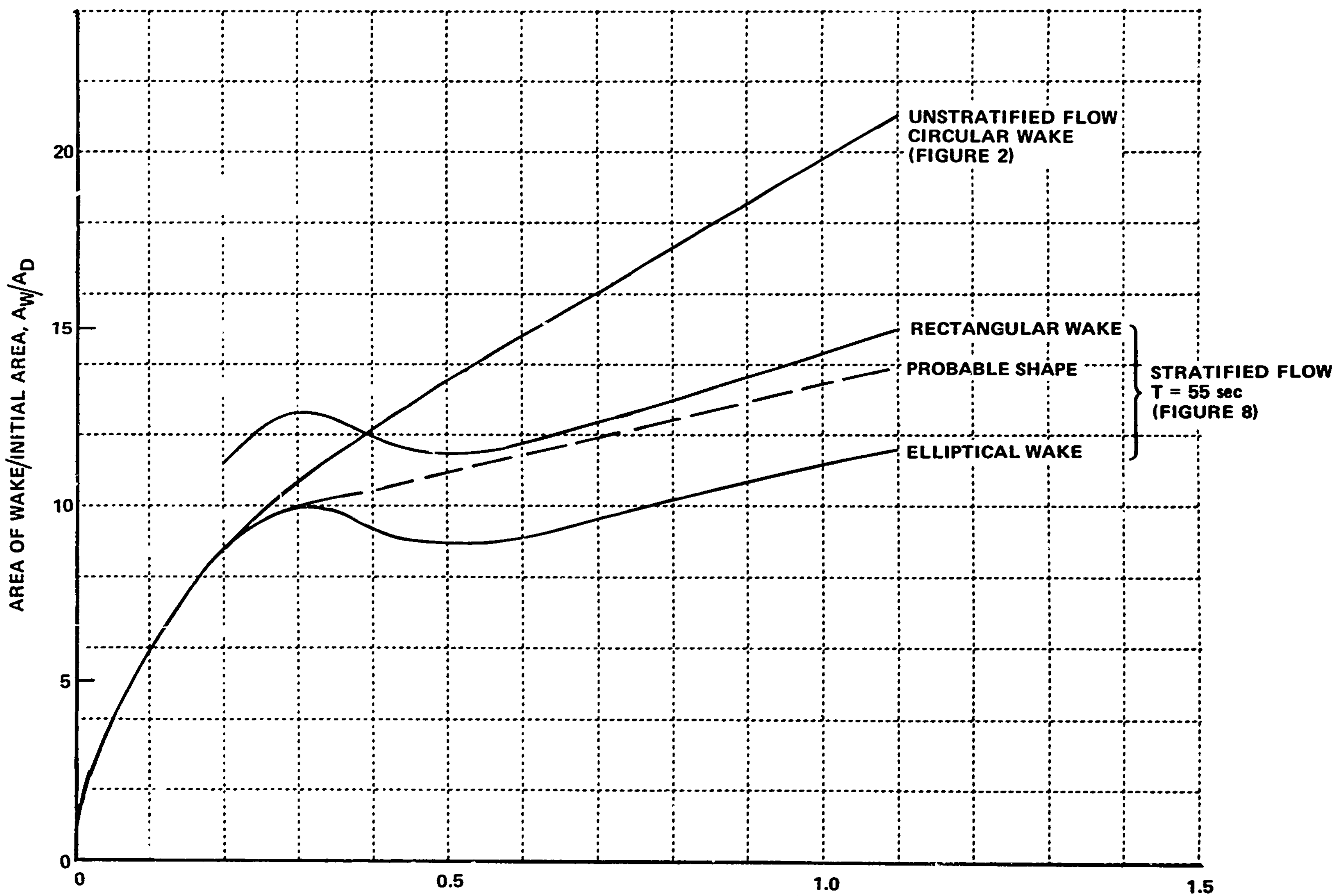

Figure 9 GROWTH OF WAKE AREA

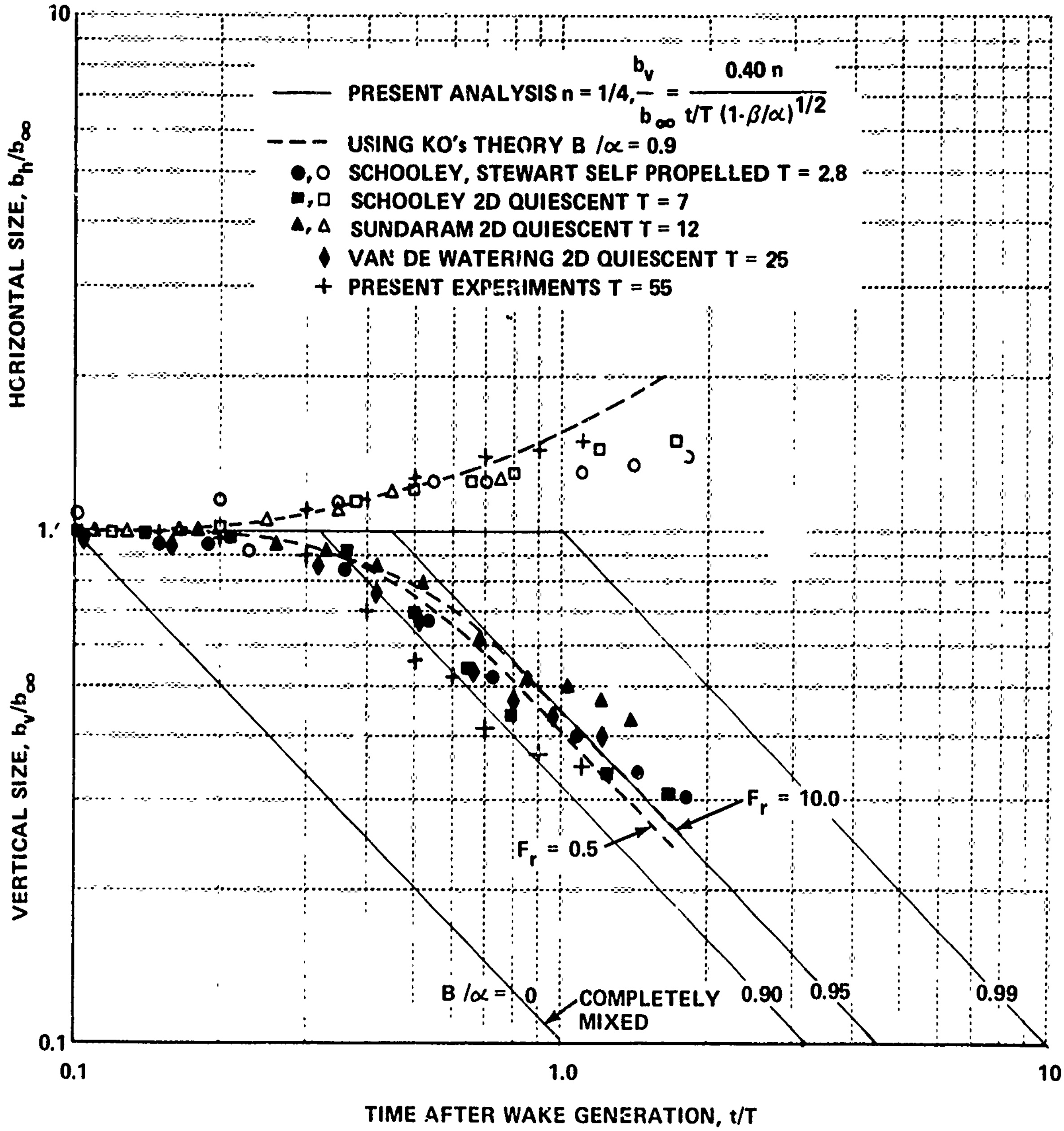

Figure 10 CORRELATION OF WAKE DIMENSIONS DURING COLLAPSE

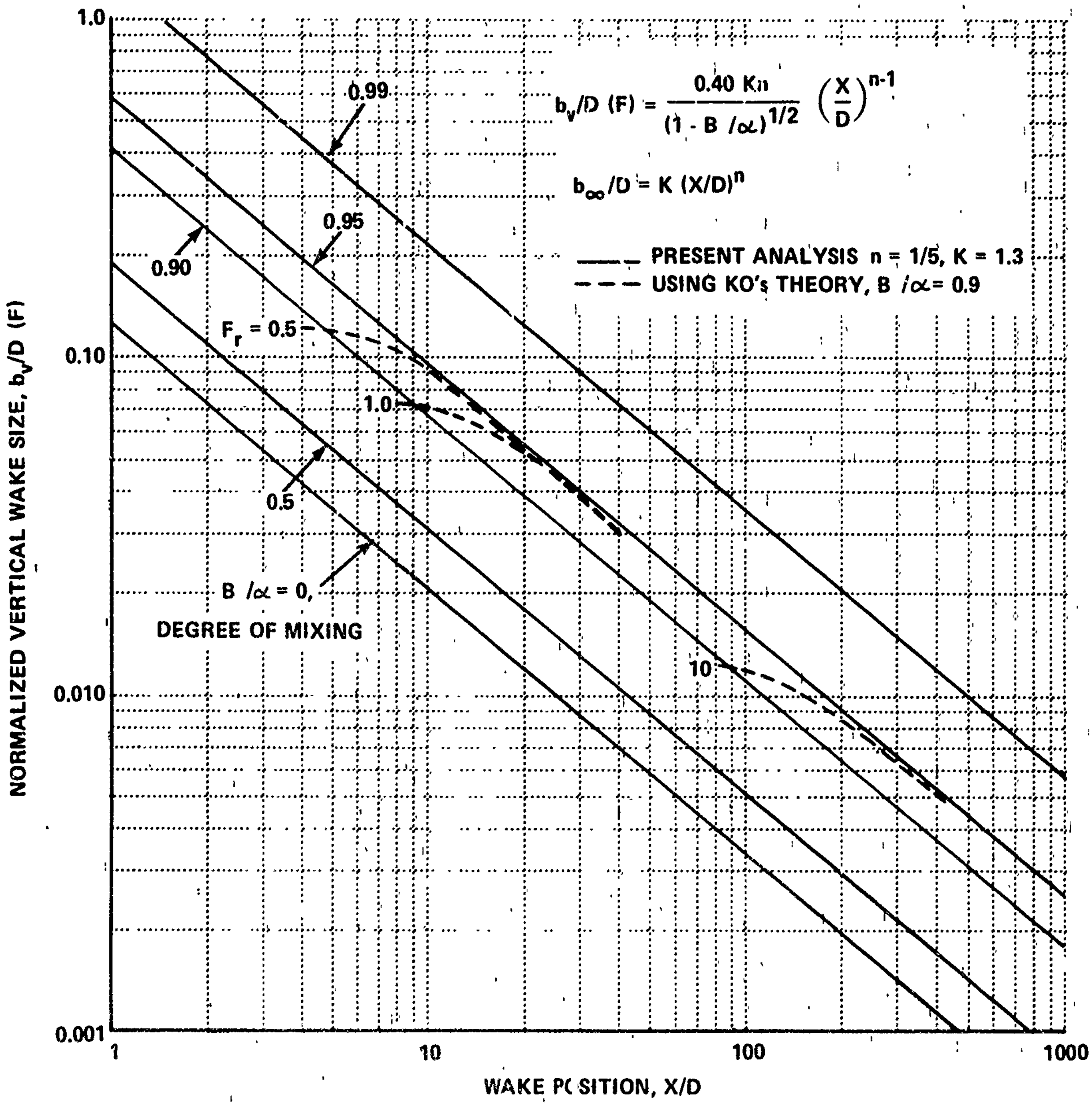

Figure 11 SCALING LAWS FOR VERTICAL SIZE DURING COLLAPSE

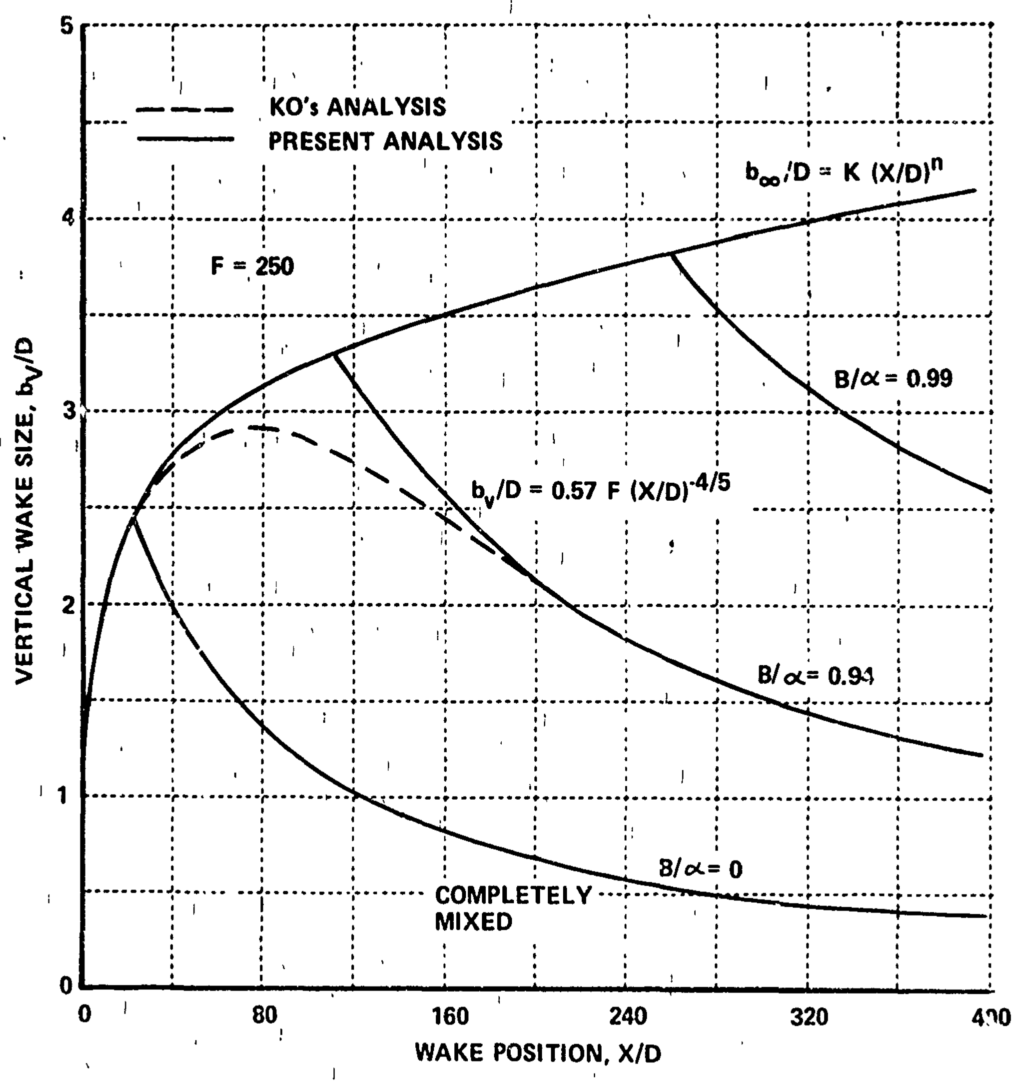

Figure 12 PREDICTIONS OF VERTICAL WAKE SIZE, F = 250

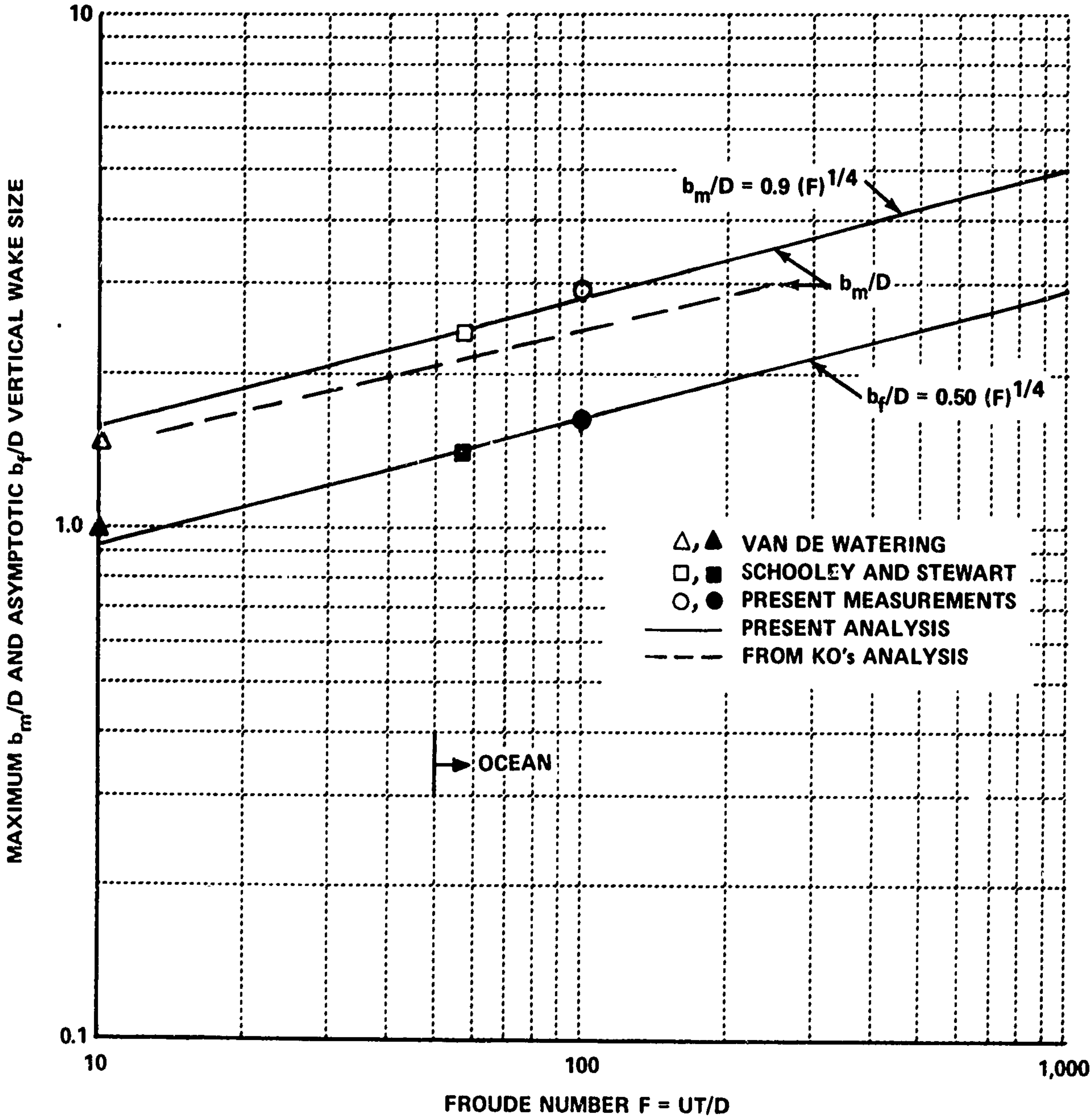

Figure 13 CORRELATION OF MAXIMUM AND ASYMPTOTIC VERTICAL WAKE SIZES

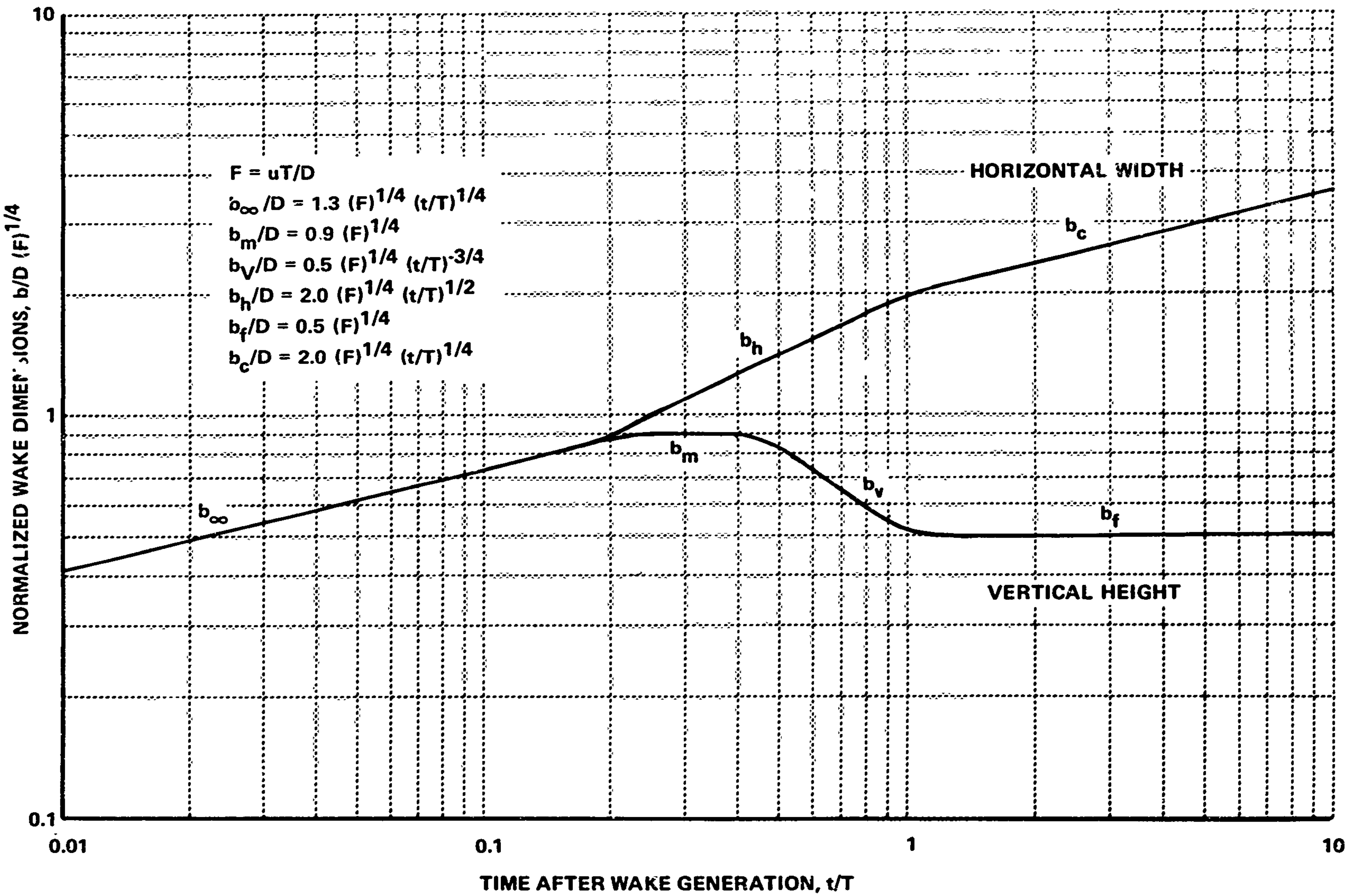

Figure 14 SCALING LAWS FOR WAKE GROWTH AND COLLAPSE

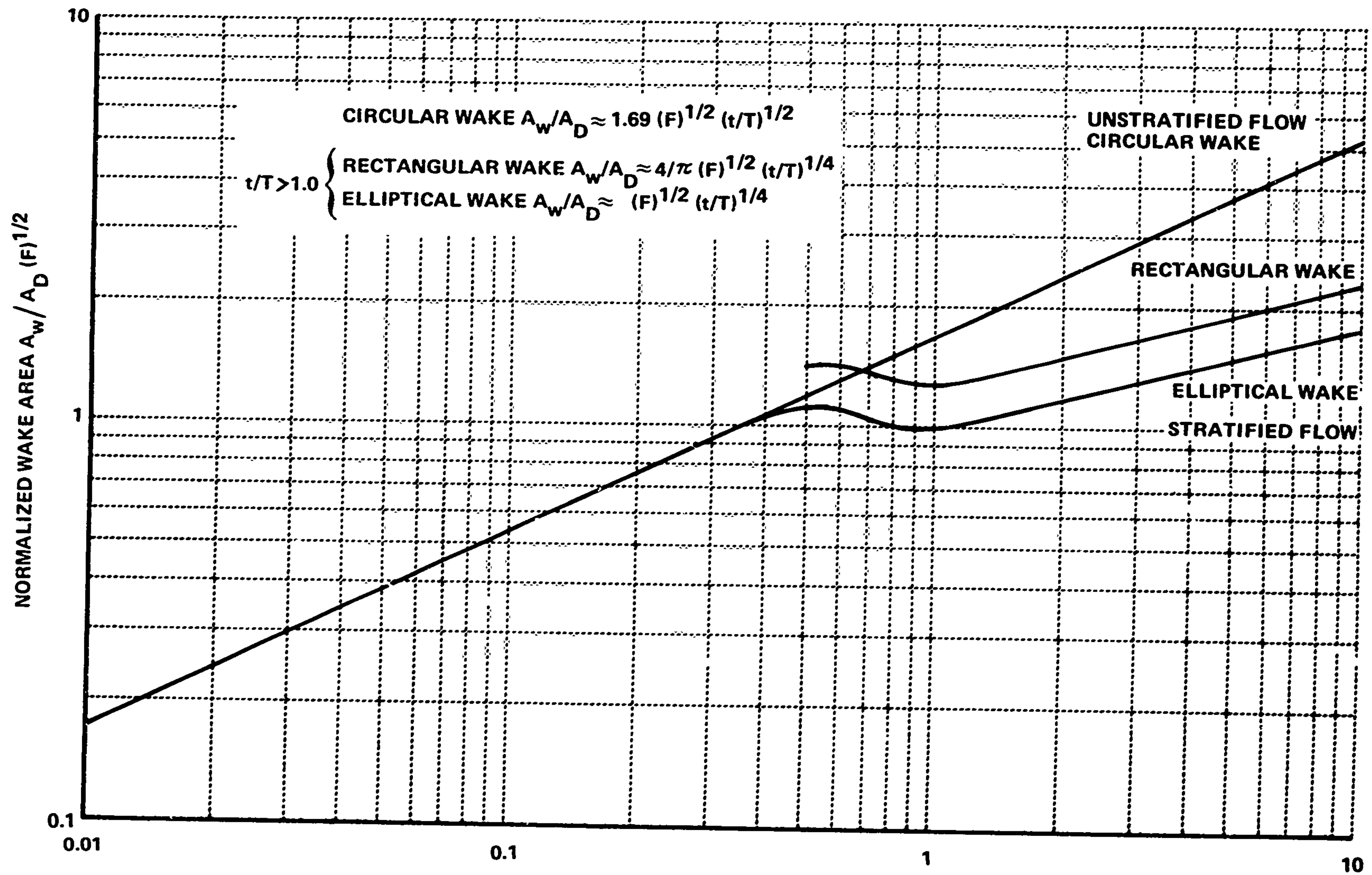

Figure 15 SCALING LAWS FOR WAKE AREA

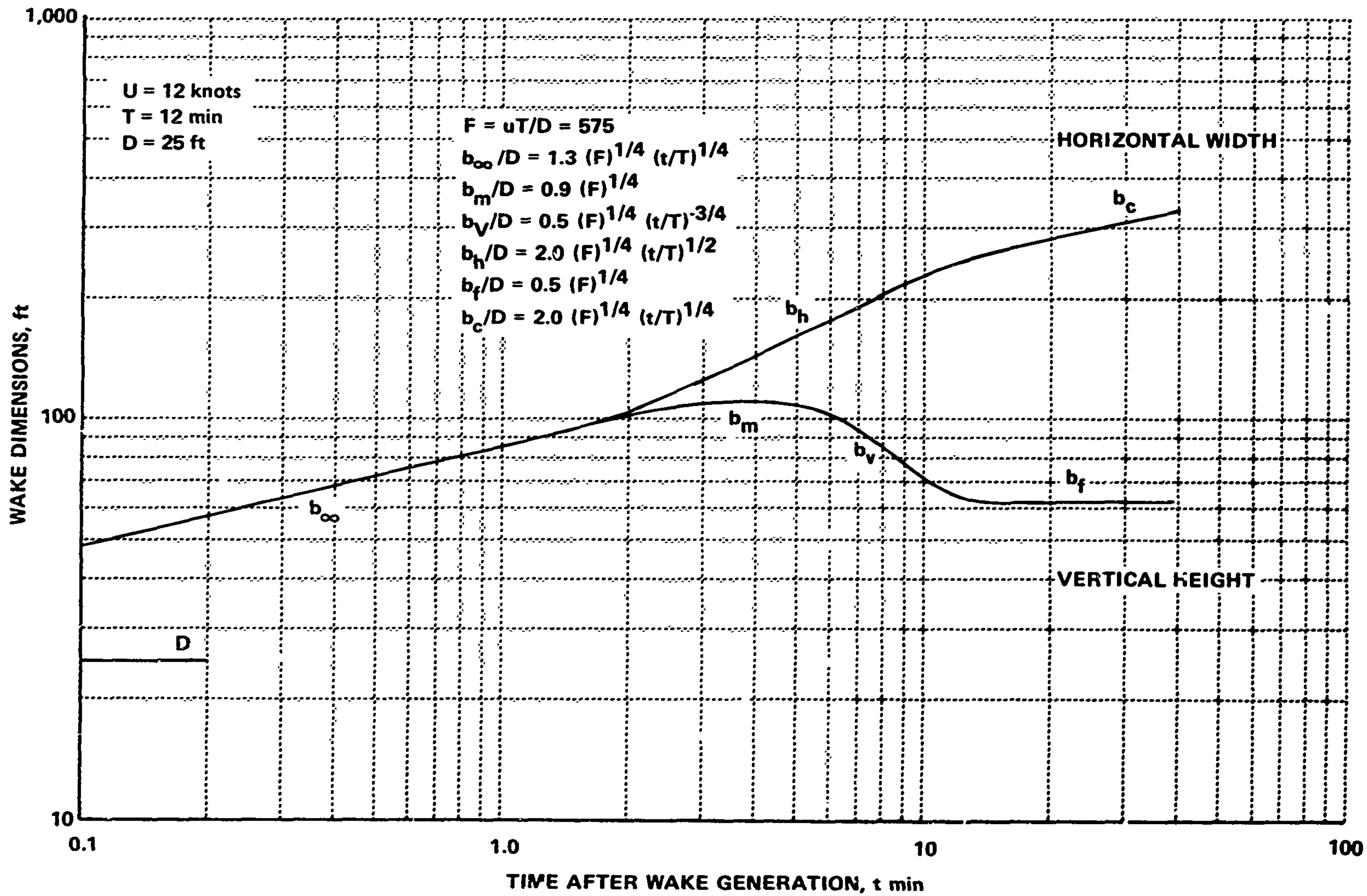

Figure 16 ESTIMATES OF FULL SCALE WAKE SIZE IN OCEAN

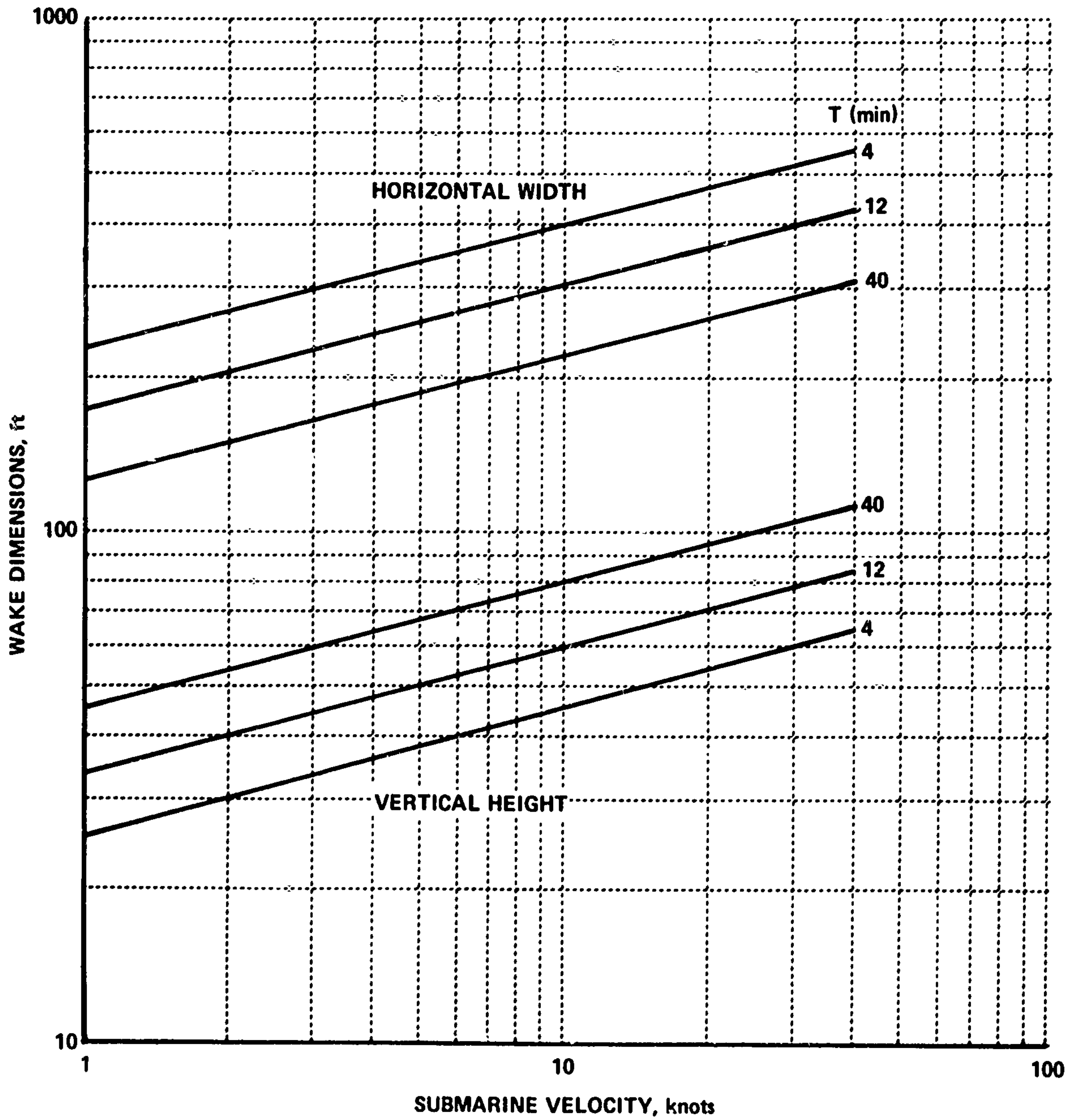

Figure 17 ESTIMATES OF WAKE SIZE AT DIFFERENT VELOCITIES AND STRATIFICATION, t = 40 min, D = 25 ft